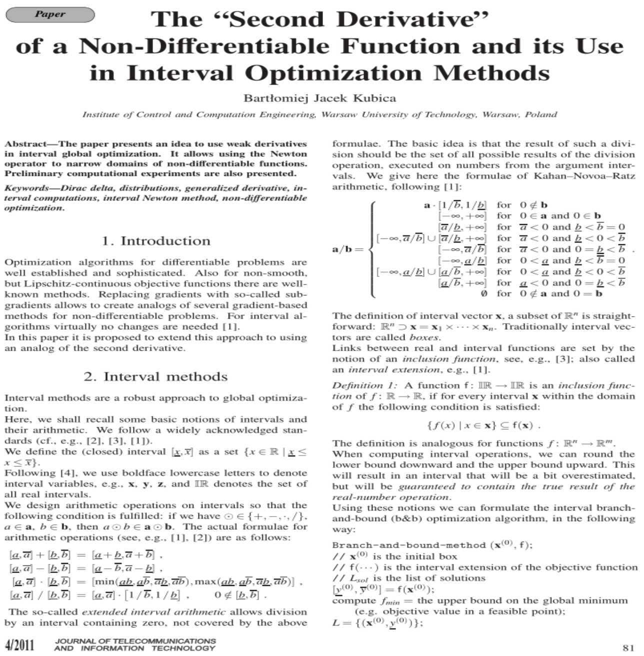
Theory And Applications Of Differentiable Functions Of Several Variables: Collection Of Papers



Paper

The "Second Derivative" of a Non-Differentiable Function and its Use in Interval Optimization Methods

Bartłomiej Jacek Kubica

Institute of Control and Computation Engineering, Warsaw University of Technology, Warsaw, Poland

Abstract—The paper presents an idea to use weak derivatives in interval global optimization. It allows using the Newton operator to narrow domains of non-differentiable functions. Preliminary computational experiments are also presented.

Keywords—Dirac delta, distributions, generalized derivative, interval computations, interval Newton method, non-differentiable optimization.

1. Introduction

Optimization algorithms for differentiable problems are well established and sophisticated. Also for non-smooth, but Lipschitz-continuous objective functions there are well-known methods. Replacing gradients with so-called sub-gradients allows to create analogs of several gradient-based methods for non-differentiable problems. For interval algorithms virtually no changes are needed [1].

In this paper it is proposed to extend this approach to using an analog of the second derivative.

2. Interval methods

Interval methods are a robust approach to global optimization.

Here, we shall recall some basic notions of intervals and their arithmetic. We follow a widely acknowledged standards (cf., e.g., [2], [3], [1]).

We define the (closed) interval $[\underline{x},\overline{x}]$ as a set $\{x \in \mathbb{R} \mid \underline{x} \le x \le \overline{x}\}$.

Following [4], we use boldface lowercase letters to denote interval variables, e.g., $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, and $\mathbb{IR}$ denotes the set of all real intervals.

We design arithmetic operations on intervals so that the following condition is fulfilled: if we have $\odot \in \{+,-,\cdot,/\}$, $a \in \mathbf{a}$, $b \in \mathbf{b}$, then $a \odot b \in \mathbf{a} \odot \mathbf{b}$. The actual formulae for arithmetic operations (see, e.g., [1], [2]) are as follows:

$$\begin{aligned}
[\underline{a},\overline{a}] + [\underline{b},\overline{b}] &= [\underline{a}+\underline{b},\overline{a}+\overline{b}] \ ,\\
[\underline{a},\overline{a}] - [\underline{b},\overline{b}] &= [\underline{a}-\overline{b},\overline{a}-\underline{b}] \ ,\\
[\underline{a},\overline{a}] \cdot [\underline{b},\overline{b}] &= [\min(\underline{a}\underline{b},\underline{a}\overline{b},\overline{a}\underline{b},\overline{a}\overline{b}),\max(\underline{a}\underline{b},\underline{a}\overline{b},\overline{a}\underline{b},\overline{a}\overline{b})] \ ,\\
[\underline{a},\overline{a}] \ / \ [\underline{b},\overline{b}] &= [\underline{a},\overline{a}] \cdot [1/\overline{b}, 1/\underline{b}] \ , \qquad 0 \notin [\underline{b},\overline{b}] \ .
\end{aligned}$$

The so-called *extended interval arithmetic* allows division by an interval containing zero, not covered by the above formulae. The basic idea is that the result of such a division should be the set of all possible results of the division operation, executed on numbers from the argument intervals. We give here the formulae of Kahan–Novoa–Ratz arithmetic, following [1]:

$$\mathbf{a}/\mathbf{b} = \begin{cases}
\mathbf{a}\cdot[1/\overline{b},1/\underline{b}] & \text{for } 0 \notin \mathbf{b}\\
[-\infty,+\infty] & \text{for } 0 \in \mathbf{a} \text{ and } 0 \in \mathbf{b}\\
[\overline{a}/\underline{b},+\infty] & \text{for } \overline{a} < 0 \text{ and } \underline{b} < \overline{b} = 0\\
[-\infty,\overline{a}/\overline{b}] \cup [\overline{a}/\underline{b},+\infty] & \text{for } \overline{a} < 0 \text{ and } \underline{b} < 0 < \overline{b}\\
[-\infty,\overline{a}/\overline{b}] & \text{for } \overline{a} < 0 \text{ and } 0 = \underline{b} < \overline{b}\\
[-\infty,\underline{a}/\underline{b}] & \text{for } 0 < \underline{a} \text{ and } \underline{b} < \overline{b} = 0\\
[-\infty,\underline{a}/\underline{b}] \cup [\underline{a}/\overline{b},+\infty] & \text{for } 0 < \underline{a} \text{ and } \underline{b} < 0 < \overline{b}\\
[\underline{a}/\overline{b},+\infty] & \text{for } \underline{a} < 0 \text{ and } 0 = \underline{b} < \overline{b}\\
\emptyset & \text{for } 0 \notin \mathbf{a} \text{ and } 0 = \mathbf{b}
\end{cases} .$$

The definition of interval vector $\mathbf{x}$, a subset of $\mathbb{R}^n$ is straightforward: $\mathbb{R}^n \supset \mathbf{x} = \mathbf{x}_1 \times \cdots \times \mathbf{x}_n$. Traditionally interval vectors are called *boxes*.

Links between real and interval functions are set by the notion of an *inclusion function*, see, e.g., [3]; also called an *interval extension*, e.g., [1].

Definition 1: A function $\mathsf{f}\colon \mathbb{IR} \to \mathbb{IR}$ is an *inclusion function* of $f\colon \mathbb{R} \to \mathbb{R}$, if for every interval $\mathbf{x}$ within the domain of f the following condition is satisfied:

$$\{f(x) \mid x \in \mathbf{x}\} \subseteq \mathsf{f}(\mathbf{x}) \ .$$

The definition is analogous for functions $f\colon \mathbb{R}^n \to \mathbb{R}^m$.

When computing interval operations, we can round the lower bound downward and the upper bound upward. This will result in an interval that will be a bit overestimated, but will be *guaranteed to contain the true result of the real-number operation*.

Using these notions we can formulate the interval branch-and-bound (b&b) optimization algorithm, in the following way:

```
Branch-and-bound-method (x^(0), f);
// x^(0) is the initial box
// f(···) is the interval extension of the objective function
// L_sol is the list of solutions
[y^(0), ȳ^(0)] = f(x^(0));
compute f_min = the upper bound on the global minimum
    (e.g. objective value in a feasible point);
L = {(x^(0), y^(0))};
```


Abstract. This collection of papers deals with investigations into various problems in the theory of differentiable functions of several variables and its applications.Collection of Papers: Dedicated to Academician Sergei Mikhailovich by the seminar on the theory of differentiable functions of several variables that has now .Collection of Papers S. M. Nikol'skii investigations in various topical trends in the theory of differentiable functions of several real variables and its applications.Collection of Papers S M Nikol. 1 94 (1) Institute of Mathematics , Issue 4 ON BEST APPROXIMATION BY SPLINES OF FUNCTION CLASSES ON THE.This article is cited in 64 scientific papers (total in 65 papers) application in the theory of differentiable functions of several variables, Collection of articles.Theory and Applications of Differentiable Functions of Several Variables: IX CORRECTION TO THE PAPER DESCRIPTIONS OF FUNCTIONS FROM.Here we study the space of functions defined on a domain with nonsmooth (Lipschitz) boundary and having derivatives with respect to all variables up to order In the same paper the case of degeneration of the weight on a plane of arbitrary.a continuous but nowhere differentiable function, yet Takagi's diverse areas of mathematics as number theory, combinatorics, and . T. Tsujii [78] constructed a Takagi-like function of two variables, and ily with applications, and papers discussing various generalizations and group of units of Z/qZ.pn\ there is also a proof in [7], These papers give different variants of the [1],) of infinitely differentiable functions on R" that are 2ir-periodic in xy for j e e.representing a real-valued function of a real variable in this calculus, we are able to In the past thirty years, motivated by applications in control theory and op- timisation the two outputs of the conditional are both differentiable: Suppose we have In this paper, instead of the test (0functions minimum.The theory of convex functions is part of the general subject of In a memorable paper dedicated to the Brunn-Minkowski to a subgroup of the linear group. Alexandrov's famous result on the second differentiability of convex functions, Convex functions of several variables: Sections where d is the generator of a one-parameter group of '-automorphisms of a C* - algebra (or just a closed . ability is usually absorbed in the theory of functions of (one or several) variable has little connection with analytic function theory; even The contents of this paper is an expanded version of a lecture delivered.Contributions to Analysis: A Collection of Papers Dedicated to Lipman Bers questions in the theory of holomorphic functions of one or several variables, the Boundary Correspondence under Conformal Mapping with Application to On ?-Monogenic Functions, and the Mean Value Theorem of the Differential Calculus.The theory of analytic functions in several variables has been concerned with theory in the circle can be made to depend on group properties of the circle, and second section of this paper is devoted mainly to a generalization of SzegS's .. The first application of Theorem 1 is a partial generalization of Jensen's formula.The theory of functions of several complex variables is the branch of mathematics dealing with The celebrated paper GAGA of Serre pinned down the crossover point from geometrie analytique to geometrie algebrique.

(respectively the Weil restriction from a totally real number field of GL(2), and the symplectic group).Vector space calculus is treated in two chapters, the differential calculus in sary purely algebraic theory of vector spaces, Chapter 4 presents the material mapping fixed-point theorem as our basic approach to the in1plicit-function . sentences occurring in mathematics contain variables and are therefore not true or false.In mathematics, the total variation identifies several slightly different concepts, related to the (local or global) structure of the codomain of a function or a measure. For a real-valued continuous function f, defined on an interval [a, b] ? ?, The extension of the concept to functions of more than one variable however is not.

[PDF] Harmonic Analysis In Phase Space
[PDF] Dare Me
[PDF] The Colonial And Post-colonial Experience: Five Centuries Of Spanish And Portuguese America
[PDF] The Nature And Dignity Of Love
[PDF] The Cambridge Introduction To Zora Neale Hurston
[PDF] The Secret Saga Of Five-sack
[PDF] Archaeological Displays And The Public: Museology And Interpretation